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14. ABSTRACT

The main objective of the proposed research is to improve various stability and handling qualities (such as Roll Over stability, Lane Change maneuver stability etc) of military, off road, unmanned multi-body ground vehicles within as wide operating envelope as possible under the presence of uncertainty in vehicle model parameters as well as under various sensor, actuator and component faults. Armed with important and award winning theoretical tools in linear and nonlinear uncertain systems theory developed by the PI, in this research project, we attempt to

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Robust Stability and Control of Multi-Body Ground Vehicles with Uncertain Dynamics and Failures

ABSTRACT

The main objective of the proposed research is to improve various stability and handling qualities (such as Roll Over stability, Lane Change maneuver stability etc) of military, off road, unmanned multi-body ground vehicles within as wide operating envelope as possible under the presence of uncertainty in vehicle model parameters as well as under various sensor, actuator and component faults. Armed with important and award winning theoretical tools in linear and nonlinear uncertain systems theory developed by the PI, in this research project, we attempt to design advanced and sophisticated control systems to improve the roll over stability and other performance objectives of multi-body ground vehicles over as wide range of speed and terrain characteristics as possible and to improve the stability and performance characteristics of these vehicles under various faults.The analysis and design algorithms resulting from these sound theoretical bases are then applied to multi-body ground vehicle stability and control problem with military vehicle application.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

R.K.Yedavalli: " Robust Control Design for Linear Systems using an Ecological Sign Stability Approach", AIAA Journal of Guidance, Control and Dynamics, Vol 32, # 1, Jan-Feb 2009, pp 348-352

R.K.Yedavalli and N. Devarakonda: " Sign Stability of Ecology and its use in Control Design with Aerospace Applications", AIAA Journal of Guidance, Control and Dynamics, Vol 33, # 2, Mar-April 2010, pp 333-346

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(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

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R.K.Yedavalli: "Robust Control Design for Roll Over Stability of Off Road, Multi-Body Ground Vehicles with Uncertain Dynamics"
Paper presented at the 20th IAVSD Symposium to be held in University of California, Berkeley, Aug 13-17, 2007

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H.H.Huang and R.K.Yedavalli,"Active Roll Control for Rollover Prevention of Heavy Articulated Vehicles with Multiple Rollover Index Minimization", Proceedings of the ASME Dynamic Systems and Control Conference, Boston, MA, Sept 2010

R.K. Yedavalli,"Robust Stability of Linear Interval Parameter Matrix Family Problem Revisited with Accurate Representation and Solution", Proceedings of the American Control Conference, June 2009, St.Louis, MO, pp 3710-3717

R.K. Yedavalli, and N. Devarakonda ,"Qualitative Principles of Ecology and their Implications in Quantitative Engineering Systems", Proceedings of the ASME Dynamic Systems and Control Conference, Hollywood, CA, Sept 2009, DSCC2009-2621

R.K. Yedavalli, and H.H.Huang ,"Controller Design for Multi-Body Ground Vehicle Rollover Prevention using Modified LQR Framework", Proceedings of the ASME Dynamic Systems and Control Conference, Ann Arbor, MI, October 2008, DSCC
2008-2230

R.K. Yedavalli, "A Necessary and Sufficient Vertex Solution for Checking Robust Stability of Interval Parameter Matrices" Proceedings of the AIAA Guidance, Navigation, and Control Conference, Honolulu, HI, August 2008, paper # AIAA 2008-7007

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(d) Manuscripts

R.K.Yedavalli and H.H. Huang,"Extension to Linear Quadratic Regulator Design Method for Control Coupled Output Regulation in Linear Systems", Paper under review for AIAA Journal of Guidance, Control and Dynamics, April 2010

H.H. Huang,"Controller Selection for State Derivative Induced Output Regulation in Linear Systems", Paper under review for the ASME Journal of Dynamic Systems, Measurement and Control, June 2010

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Patents Submitted

Patents Awarded

Graduate Students

NAME	PERCENT_SUPPORTED
Hsun Hsuan Huang	0.50
Nagini Devarakonda	0.50
FTE Equivalent:	**1.00**
Total Number:	**2**

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Rama K. Yedavalli	0.10	No
FTE Equivalent:	**0.10**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

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Names of Personnel receiving masters degrees

NAME	
Total Number:	

Names of personnel receiving PHDs

NAME	
Hsun Hsuan Huang	
Total Number:	**1**

Names of other research staff

NAME	PERCENT_SUPPORTED
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Final Report– Grant # 49461-EG

Robust Stability and Control of Multi-body Ground Vehicles with Uncertain Dynamics and Failures

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Chapter 1. Overview of the Research Project

1.1 Objectives

The main objective of this research was to improve various stability and handling qualities (such as Roll Over stability, Lane Change maneuver stability etc) of military, off road, unmanned multi-body ground vehicles within as wide operating envelope as possible under the presence of uncertainty in vehicle model parameters as well as under various sensor, actuator and component faults. Armed with important and award winning theoretical tools in linear and nonlinear uncertain systems theory developed by the PI, in this research project, we designed advanced and sophisticated control systems to improve the roll over stability and other performance objectives of multi-body ground vehicles over as wide range of speed and terrain characteristics as possible and to improve the stability and performance characteristics of these vehicles under various faults.

1.2 Approach

The research approach was to first model the multi-body ground vehicle such as FTTS (Future Tactical Truck System) and HTV (Heavy Tactical Vehicle) as an integrated system consisting of various subsystems (individual component bodies, each with its own dynamic characteristics) . Then the next task was to identify the critical parameters that are responsible for significant effect on the vehicle stability (such as roll over stability). Then, a careful analysis was carried out for various steering inputs such as steady turn, lane change and any other maneuvering inputs. Similarly, the effect of parameters like road surface coefficient μ (denoting the condition of the road, namely icy, dry etc) and c_f and c_r (the cornering stiffness coefficients) were studied. To accomplish all these tasks, various innovative theoretical frameworks were developed. One of them can be labeled as `Extended Coupled LQR (Linear Quadratic Regulator) Control Design Technique' in which a control coupled output is regulated and properties of resulting controller are fully exploited to design controllers of reduced control effort for a given performance objective. Then to accommodate uncertainty in the design, two additional innovative theoretical frameworks were developed. One of them can be labeled as

`advanced robustness analysis and design techniques for interval parameter dynamic systems' and the other labeled as `ecological sign stability' approach. This latter approach is relatively new and promising. The analysis and design algorithms resulting from these sound theoretical bases are then applied to multi-body ground vehicle stability and control application clearly illustrating the efficacy of these approaches.

1.3 Relevance to Army

Future army vehicles such as Future Tactical Truck System (FTTS) and Heavy Tactical Vehicle (HTV) are multi-body ground vehicles which are required to perform different maneuvers at various operating conditions with high degree of mobility and stringent requirements on their handling. The research being carried out under this project is deemed to enhance the stability and control characteristics for military, off road, unmanned multi-body ground vehicles such as these under adverse weather and terrain conditions as well as under component faults. This research is useful to improve the performance of next generation military vehicles to achieve optimal balance of payload, mobility and survivability.

1.4 Project Significance

The approaches researched under this project sponsorship are innovative in the sense that the realistic scenario of the performance requirements of off road, military ground vehicles are captured in sound theoretical frameworks of `extended coupled LQR', `interval parameter systems' and `ecological sign stability' and as such solutions are sought from a `systems' level view point. These powerful theoretical techniques, when applied to military multi-body ground vehicle dynamics problems result in guaranteed stability and performance improvements in these vehicles. This type of `systems level' approach in this research allows the results obtained under this project sponsorship to not only benefit the army applications but other applications of relevance to air force, navy, and automotive industry as well.

1.5 Accomplishments for the Project Period

Research accomplishments for this project comprise the development of various dynamic models in state space framework of increasing complexity for multi-body ground vehicles and continued progress on the theoretical development of robust stability analysis of linear interval parameter state space models. Efforts to understand the role being played by various components such as suspension subsystem, wheel subsystem, relative mass and center of gravity distribution of the component bodies etc are promising and rewarding. A major innovative approach, developed during this project period is to design nominal controllers based on `extended coupled Linear Quadratic Regulator' technique. In this technique, the control coupled output is regulated in an LQR (Linear Quadratic Regulator) framework, clearly delineating the coupling effect between the control and output which in turn is exploited to design controllers of reduced control effort for a given performance objective. Simultaneously, to address and accommodate the issue of uncertainty in the models, research was carried out on designing robust controllers. Significant progress was made in designing robust controllers using the

promising `ecological sign stability' approach. Thus we investigated two approaches of robust control design: one based on traditional `interval parameter theory' and the other based on `ecological sign stability' theory. Of course, it turns out that both of these approaches are in a way related and complement each other. Research based on these two approaches is quite promising and are yielding results which are very beneficial in finding solutions to the stability and control problems of off road, multi-body ground vehicles. In a nut shell, the accomplishments for this project period can be summarized as follows.

- Development of Various State Space Models of increasing complexity to capture Roll Over phenomenon and Other Performance and Ride Quality requirements. In this connection, first simple `Bicycle' models are considered. Then two axis (that include Roll and Yaw motions) models are considered. Then higher order models that include all three degrees of freedom (Roll, Yaw and Pitch) are considered. These higher order, more sophisticated and involved models capture the realistic scenario as closely as possible. In addition, we developed models that include the terrain (road) profile, and flexible active suspension sub systems with torsional flexibility as well.

- A new, innovative technique is developed in this project period in which the traditional `Roll Over Index' reported in the literature is modified in such a way that it can be directly used in the optimal control design procedure such as the Linear Quadratic Regulator (LQR) design. The novelty of the approach lies in expressing the traditional rollover index as a quadratic performance index directly and explicitly in the optimization procedure. The control gain obtained based on the proposed new rollover performance index is more effective for rollover prevention than the gain obtained based on standard LQR design in which there is too much ambiguity and labor involved in assigning the weights and the new procedure makes the weight selection more systematic and straightforward. The properties of this `extended coupled LQR' method are thoroughly analyzed which in turn allow us to identify system dynamics for which controllers of reduced control effort can be designed for a given output regulation objective.

- Development of `Interval Parameter Robust Control' theory to analyze and design robust controllers with the parameters of the above models taken as uncertain and thus assumed to vary within given intervals with lower and upper bounds. The significant progress in this task is to be able to obtain explicit expressions for the convex combination coefficients in terms of the uncertain parameters (which was never done in the literature till now). This capability is expected to yield improved analysis algorithms for checking robust stability of matrix families, which in turn are very useful for our multi-body vehicle dynamics models.

- Substantial insight is provided by identifying vehicle forward velocity as a critical parameter in expressing the dynamics of the multi-body ground vehicle. With this as the critical parameter, which varies nonlinearly in the plant matrix, a linear parameter varying model is obtained by `overbounding' this parameter, taking care to reduce the

conservatism as much as possible. Then the linear interval parameter matrix family stability theory is then successfully applied to this `overbounding' parameter model.

- In this research, we developed a `control design' procedure for linear interval parameter dynamic systems. When this robust control design procedure is applied, the simulation of a double lane change maneuver shows that the controller not only decreases roll overshoot of the vehicle's body during lane change and/or obstacle avoidance maneuvers but also decreases the rollover risk in the transient also.

- We also developed robust controllers using `Ecological Sign Stability Theory'. `Robustness' refers to system's behavior under perturbations in the plant (system/process) parameters. This new, innovative idea is attracting considerable attention from the peer research community. With this procedure, it is possible to come up with a single controller that performs satisfactorily not only for the vehicle it is designed for but also performs well for an entire class of vehicles without any need for redesign, resulting in substantial savings in development costs.

- In this project period, attention is also paid to the issue of designing controllers for maintaining the stability and performance characteristics of the vehicle under various faults. These faults can be classified as input (actuator) faults, output (sensor) faults and parameter (component) faults. The input (actuator) fault can be detected and isolated using unknown input observer (UIO). The output (sensor) fault can be detected and isolated using Dedicated observer scheme (DOS) (observer-based residual generation). A strategy for designing a controller to overcome the parameter (component) faults is being developed based on interval polytope framework.

1.6 Collaborations and Technology Transfer

Subsequent to the formal meeting PI had with Mr. Mike Letherwood and his research group at TARDEC/TACOM in April'07, at which the PI gave a formal presentation about the ongoing research to the group, continued dialogue between the two groups resulted in the PI attending the SAE World Congress in April'08 as an invitee of the First APBA (Advanced Planning Briefing for Academia) organized by TARDEC . In that meeting, ground work was laid to have formal technology transfer between Dr. Yedavalli's research group and TARDEC. To make further progress on this technology transfer issue, the PI again made a trip to TARDEC/TACOM in Warren, MI on June 26^{th} '08 to meet with Mr. David D. Gunter and Mr. JamesO'Kins. In that meeting it was decided that Dr. Yedavalli's group would supply the `robust control design algorithm' developed under this research project to the TARDEC/TACOM group which will then be tested by that group for their vehicle models. It was also agreed upon that the TARDEC/TACOM group would supply the OSU group with generic, but representative dynamic model parameters of vehicles of interest to TARDEC/TACOM so that Dr. Yedavalli's group can work on more realistic data of relevance to army vehicles. In March 2009, the control design algorithm code was supplied to TARDEC group and we are currently waiting for their response and feedback.

In addition, the PI, Dr. Yedavalli is making progress with the Ohio State University Technology Licensing Office on the process of establishing a small business, registered as `Robust Engineering Systems, LLC' that develops, services and sells a software code package labeled `ASSURES' (Analysis, Synthesis and SimUlation of Robust Engineering Systems'). The research results obtained under this present ARO supported research project will partially form the basis for the development of this package.

In addition, Ms. Nagini Devarakonda, currently a Ph.D student working with Dr. Yedavalli under this research project, was awarded the competitive and prestigious `Internship' position at the General Motors (GM) Research and Development office, in Warren, MI for the Summer of 2008.

Similarly, Ms. Hsun-Hsuan Huang, who graduated with a Ph.D degree under this research project, was invited and interviewed by a University of Notre Dame research group to present her research results of this project. Currently, she is seeking an employment.

Chapter 2. Details of Technical Contributions

2.1 Rollover Prevention of Multi-body Ground Vehicles Extending LQR design for Control Coupled Output Regulation

2.1.1 Abstract

In this paper, a novel control system analysis and design technique is presented by transforming rollover index minimization as an optimal control problem in the popular Linear Quadratic Regulator (LQR) control design framework. This results in a 'control coupled output regulation' problem because of the coupled terms in state and control variables in the performance index. There are no guidelines available for the selection of state and control weightings for the coupled case. In this paper, we present considerable insight into the major differences in the optimal control problem formulation for the uncoupled and coupled cases and provide guidelines for weight selections by taking into consideration the interaction (or coupling) between state and control terms and using the property of 'Output Zeroing' controller. The proposed analysis and design method is illustrated with rollover prevention control for a multi-body Ground Vehicle using the active steering control and active roll control respectively.

2.1.2 Introduction

Rollover prevention is a fundamental and significant issue for vehicle dynamics and has been a topic of considerable research for a long time [1-13]. Rollovers are divided into two broad categories: tripped and un-tripped. A tripped rollover commonly occurs when a vehicle skids and digs its tires into soft soil or hits a tripping mechanism such as a curb or guardrail with a sufficiently large lateral velocity. Maneuver-induced un-tripped rollover can occur during typical driving situations and poses a real threat for the vehicles with an elevated center of gravity. Examples are excessive speed during cornering, obstacle avoidance and severe lane change maneuvers, where rollover occurs as a result

of the lateral wheel forces induced during these maneuvers. Furthermore, vehicle rollover can also occur during external disturbances like side-wind. Thus, passenger vehicles with a high center of gravity such as light trucks (vans, pickups, and SUVs (Sport Utility Vehicles)) are more prone to rollover accidents. Moreover, the heavy commercial vehicles with narrow track width are often involved in rollover accidents. Also, rollover prevention is of high importance for military vehicles, which operate in severe operational environments and maneuvers. Rollover prevention is a critical safety issue but there is still no safety performance standard available. In addition, there are no well-recognized rollover protection standards or design guidelines.

Typically, a driver does not have any indication before a rollover happens and many rollover situations cannot be prevented by driver actions alone, even when they are correctly warned. Additional assistance from active anti-rollover control can mitigate the deficiency in human capability. Hence, rollover prevention systems are classified into two stages: detection of the possibility of a rollover, and development of a mitigation control algorithm. Thus, the research on rollover prevention systems has mainly focused on two areas: rollover detection systems and anti-rollover control systems. In this paper, the emphasis is on developing a control algorithm for anti-rollover control systems.

After the risk of rollover is detected, effective anti-rollover control systems are crucial to prevent vehicles from rollover or help vehicles to recover from rollover. In recent years with the development of advanced control technology and cost reduction of electronic and control equipment, active control has been widely used in the automotive industry in the design of anti-rollover control systems. There are four major active rollover control applications based on the actuation schemes: (1) Active anti-roll-bar systems; (2) Active suspension systems (3) Anti-roll braking systems; and (4) Active steering systems. Active anti-roll-bar and active suspension systems directly control the vehicle roll motion. Anti-roll braking systems and active steering systems reduce vehicle oversteer and control vehicle yaw moment. An active anti-roll-bar hydraulically determines the variation of the equivalent stiffness of the anti-roll-bars. The vehicle load distribution is influenced by the roll-bar stiffness distribution such that the roll angle and roll moment are improved by an active anti-roll-bar. The use of an anti-roll-bar system to improve vehicle roll stability and reduce the rollover has been proposed and developed, especially for heavy road vehicles [8-12]. Active suspension systems use electrohydraulic equipment to generate controlled vertical forces to react to rollover moments and are used to gain improvements in both roll and ride performance [3,4,13]. Both anti-rollover braking system and active steering system essentially control yaw moment to reduce rollover risk indirectly due to the coupling between roll, lateral, and yaw dynamics. An anti-rollover braking system controls the front brakes to reduce the cornering capability of the front tires, which causes the vehicle to turn less sharply and reduces its speed to prevent the rollover [1,2,5,14]. Active steering systems control the steering input directly to reduce the rollover risk by reducing or reversing the steering angle to reduce or reverse the unstable roll maneuver [6,7,15,17]. The steering input significantly influences lateral vehicle dynamics, and an excessive steering command may result in unstable vehicle motion, especially for military vehicles, which operate in severe operational environments and maneuvers and can be treated as an emergency control. In this paper, a 3-Degrees Of Freedom (DOF) yaw-roll vehicle model for a Sport Utility Vehicle (SUV) in [6] with active steering

control and active roll control is considered and the proposed control design methodology is applied and results are discussed.

The paper is organized as follows: in the next section, vehicle dynamics modeling with active steering control and active roll control respectively is described. In section 2.1.4, a new unified rollover index is introduced. In section 2.1.5, a rollover performance index is proposed to be used in the optimal control formulation. In section 2.1.6, an extension to Linear Quadratic Regulator (LQR) design for control coupled output regulation for a class of systems is presented. In section 2.1.7, the control design algorithm developed in section 2.1.6 is applied to the rollover prevention problem of the 3-DOF yaw-roll vehicle model with active steering control and active roll control respectively and results are discussed. Finally, section 2.1.8 offers some concluding remarks.

2.1.3 Vehicle Model with Active Steering Control and Active Roll Control

In this paper, we consider a 3-DOF yaw-roll vehicle model in [16] and two different controllers are assumed for this model. One is active steering control and the other one is active roll control. The vehicle dynamic model in state space form is given by,

$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$, where

$\mathbf{A} = -\mathbf{E}^{-1}\mathbf{U}, \mathbf{B}_1 = \mathbf{E}^{-1}\mathbf{V}_1, \mathbf{B}_2 = \mathbf{E}^{-1}\mathbf{V}_2$,

$\mathbf{x} = [v_y \quad r \quad \dot{\phi} \quad \phi]^T, \mathbf{u}_1 = \delta$ and $\mathbf{u}_2 = M_z$. Also,

$$\mathbf{E} = \begin{bmatrix} M & 0 & M_s h & 0 \\ 0 & I_z & I_{xz} & 0 \\ Mh & I_{xz} & I_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \mathbf{U} = \begin{bmatrix} U_{11} & U_{12} & 0 & U_{14} \\ U_{21} & U_{22} & 0 & U_{24} \\ 0 & U_{32} & U_{33} & U_{34} \\ 0 & 0 & -1 & 0 \end{bmatrix};$$

$$\mathbf{V}_1 = [Y_\delta \quad N_\delta \quad 0 \quad 0]^T; \mathbf{V_2} = [0 \quad 0 \quad 1 \quad 0]^T$$

$$U_{11} = -Y_\beta / u_0; U_{12} = Mu_0 - Y_r; U_{14} = -Y_\phi;$$

$$U_{21} = -N_\beta / u_0; U_{22} = -N_r; U_{24} = -N_\phi;$$

$$U_{32} = M_s h u_0; U_{33} = -L_p; U_{34} = -L_\varphi .$$

All parameters of interest and the numerical values of the parameters of the model are taken from [16]. The states are roll angle (ϕ) of sprung mass relative to unsprung mass, lateral velocity (v_y), yaw rate (r) and roll rate ($\dot{\phi}$) of sprung mass relative to unsprung mass. The control input is steering angle (δ) or roll moment (M_z), with the corresponding control distribution matrices given by $\mathbf{B_1}$ and $\mathbf{B_2}$ respectively.

2.1.4 Rollover Index (RI)

Rollover Index is an important metric in vehicle safety assessment. A variety of rollover indices have been introduced in the literature [3]. In rollover detection systems, the concept of a rollover index is used to determine the threshold for rollover. Various rollover thresholds are derived from different factors which influence rollover such as the

position of a vehicle's center of gravity (CG), the energy of rollover, and vertical tire forces. In this paper, we develop a 'unified rollover index' using the concept of Lateral Transfer Ratio (LTR) described in [5,6] by taking the torque balance for an unsprung mass about the zero-level center. Note that once wheel lift occurs, anti-rollover control is very difficult. Therefore, the concept of LTR is selected in this research.

Rollover index is defined as the load difference (i.e., vertical force) between the inside and outside wheels of the vehicle, normalized by the total load. That is,

$$\begin{aligned}\mathrm{RI} &= \frac{\text{Load on Outside Tires} - \text{Load on Inside Tires}}{\text{Total Load}} \\ &= \frac{F_{z,o} - F_{z,i}}{F_{z,o} + F_{z,i}} = \frac{F_{z,o} - F_{z,i}}{mg}\end{aligned}$$

where g is gravity, m is mass of the total vehicle and $m = m_s + m_u$ (m_s is sprung mass and m_u is unsprung mass). With this definition of the index, the vehicle is considered 'rolled over' when the Rollover Index (RI) is equal to 1 or -1. In other words, the vehicle does not roll over as long as $|\mathrm{RI}| < 1$. For $F_{z,o} = F_{z,i}$ (i.e., RI = 0), the vehicle drives straight on a horizontal road. When $F_{z,o} = 0$ (i.e., RI = -1), the outside wheels lift off the road. When $F_{z,i} = 0$ (i.e., RI = 1), the inside wheels lift off the road.

In [14,15], the authors argued that the rollover estimation in [6] is not sufficient to detect the transient phase of rollover due to the fact that it is derived ignoring roll dynamics. Furthermore, in [14,15] the rollover index is given by $\mathrm{RI} = -\frac{2(k\phi + c\dot{\phi})}{mgT}$, where T is track width, ϕ and $\dot{\phi}$ are roll angle and roll rate of body 2 relative to body 1 respectively and k, c denote the total torsional spring stiffness and torsional damper coefficients of the suspension respectively. However, even this is also not sufficient to estimate the rollover since the lateral dynamics is ignored in the formula. The lateral acceleration is a critical factor in rollover. Therefore, in what follows, we present a new rollover index which includes the lateral and roll dynamics simultaneously.

A schematic of the vehicle rollover model is shown in Fig. 1. Body 1 is unsprung mass ($m_u = m_1$) and CG1 is CG of body 1. Body 2 is the sprung mass ($m_s = m_2$) and CG2 is CG of body 2. The unsprung mass is assumed to be insignificant and assumed to roll about a horizontal roll axis, which is along the centerline of the unsprung mass and at the ground level. Hence, it is reasonable to neglect the unsprung mass inertial contribution. The effective linear torques exerted by the suspension system about the roll center are defined as $T_{\text{spring}} = k\phi$ and $T_{\text{damper}} = c\dot{\phi}$, where ϕ and $\dot{\phi}$ are roll angle and roll rate of body 2 relative to body 1 respectively and k, c denote the total torsional spring stiffness and torsional damper coefficients of the suspension respectively. The unified rollover index is derived from the torque balance of the unsprung mass about the zero-level center, which is the point S in the Fig. 1. The result of the torque balance is shown in Eq. (1).

$$\begin{cases}(-F_{z,o} + F_{z,i})\dfrac{T}{2} + F_y h_R + k\phi + c\dot{\phi} = 0 \\ F_y = m_2 a_{y,2} = m_2(\dot{v}_y + v_x r - h\ddot{\phi})\end{cases} \tag{1}$$

$$\Rightarrow F_{z,o} - F_{z,i} = \frac{2}{T}\left(m_2 a_{y,2} h_R + k\phi + c\dot{\phi}\right) \quad (2)$$

$$= \frac{2}{T}\left[m_2(\dot{v}_y + v_x r - h\ddot{\phi})h_R + k\phi + c\dot{\phi}\right]$$

where F_y is lateral force, $a_{y,2}$ is the lateral acceleration of the CG2, h_R is distance from the roll axis to the ground, h is distance from the CG2 to the roll axis, T is track width, r is yaw rate of body 1, v_x is longitudinal velocity of body 1, $\dot{v}_y$ is the lateral acceleration of body 1, and $\ddot{\phi}$ is roll acceleration of body 2 relative to body 1. In a straight driving condition with constant forward velocity v_x, the lateral acceleration of body 2 is given by $a_{y,2} = \dot{v}_y + vr - h\ddot{\phi} = \dot{v}_y + v_x r - h\ddot{\phi}$. Hence, the rollover index can finally be expressed as

$$\mathrm{RI} = \frac{F_{z,o} - F_{z,i}}{F_{z,o} + F_{z,i}} = \frac{F_{z,o} - F_{z,i}}{mg} = \frac{2[m_2(\dot{v}_y + v_x r - h\ddot{\phi})h_R + k\phi + c\dot{\phi}]}{mgT} \quad (3)$$

Since both roll and lateral dynamics can affect rolling moment, the unified rollover index presented in the above Eq. (3), which includes the roll and lateral dynamics is more appropriate and effective in detecting rollover.

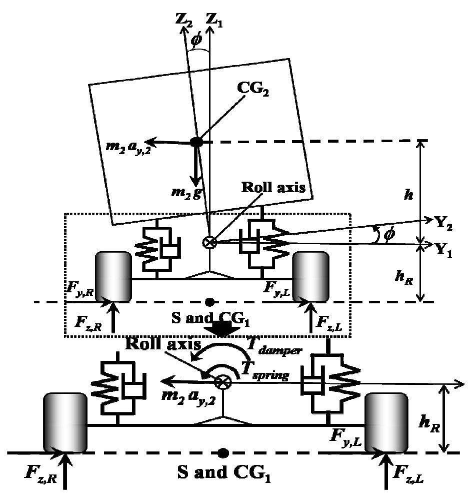

FIGURE 1 VEHICLE ROLLOVER MODEL

2.1.5 Rollover Performance Index

In [17], an approach in which the rollover index minimization is transformed into an optimal control problem in LQR framework is presented. In this section, adopting the same idea, the rollover performance index for the vehicle model with two different controllers is considered. Notice that the states for the system are roll angle (ϕ), lateral velocity (v_y), yaw rate (r) and roll rate ($\dot{\phi}$), and the control input is either steering angle (δ) or roll moment (M_z). Rollover Index is a linear combination of all states and the

control input. Hence, the output equation for the vehicle model with active steering control is

$$\mathrm{RI} = \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}_1 \delta \tag{4}$$

and the output equation for the vehicle model with active roll moment control is

$$\mathrm{RI} = \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}_2 M_z \tag{5}$$

where $\mathbf{x} = [\phi \quad v_y \quad r \quad \dot{\phi}]^T$. The output in Eq. (4) and Eq. (5) can be incorporated into the performance index of LQR framework to form the rollover performance index as discussed in [17].

Notice that in this particular application, these systems have only one input and one output and thus $\mathbf{D}_1$ and $\mathbf{D}_2$ are 1×1 square matrices (i.e. scalars) and we take advantage of this fact in the control design presented later.

2.1.6 LQR Design with Control Coupled Output Regulation in Rollover Prevention Problems

In this section, an extension to LQR design with control coupled output regulation is proposed and applied to a vehicle dynamics problem. A very popular method for control design of linear dynamic systems is the LQR method [18].

The state equation in the state space form for a linear system is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \qquad \mathbf{x}(\mathrm{t}_0)\ \text{given} \tag{6}$$

where **A** is n by n matrix; **B** is n by m matrix; **x** is n by 1vector; **u** is m by 1vector;

The performance output equation in the state space form for the standard (uncoupled) output regulation problem is represented by

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{7}$$

However, in this paper, we address the control coupled output regulation problem where the output is given by

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \tag{8}$$

where **C** is k by n matrix; **D** is k by m matrix; **y** is k by 1vector and the situation gets slightly more complicated and interesting. That is, when the output **y** is coupled to the control, the typical tradeoff described for the uncoupled case between output regulation cost and control effort [19] is no longer that straightforward due to the coupling. Note that in the control coupled output regulation problem, output regulation is no longer equivalent to pure state regulation problem. This rollover performance index in section 2.1.5 shows the LQR design for the rollover prevention problem belongs to the LQR design for the control coupled output problem.

Recalling the Performance Index in LQR design for control coupled output regulation problem which is denoted by J, written in terms of the output equation $\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$, we have

$$
\begin{aligned}
J &= \int_{t_0}^{\infty} \left[\mathbf{y}^T(\tau)\overline{\mathbf{Q}}\mathbf{y}(\tau) + \rho\,\mathbf{u}(\tau)^T\,\overline{\mathbf{R}}\mathbf{u}(\tau)\right] d\tau \\
&= \int_{t_0}^{\infty} \left[\mathbf{x}^T(\tau)\mathbf{Q}\mathbf{x}(\tau) + 2\mathbf{x}^T(\tau)\mathbf{N}\mathbf{u}(\tau) + \mathbf{u}^T(\tau)\mathbf{R}\mathbf{u}(\tau)\right] d\tau
\end{aligned} \tag{9}
$$

where $\mathbf{Q} = \mathbf{C}^T\,\overline{\mathbf{Q}}\mathbf{C}$; $\mathbf{N} = \mathbf{C}^T\,\overline{\mathbf{Q}}\mathbf{D}$; and $\mathbf{R} = \mathbf{D}^T\,\overline{\mathbf{Q}}\mathbf{D} + \rho\overline{\mathbf{R}}$. The weighting matrix $\mathbf{Q}$ is a symmetric positive semi-definite and the weighting matrix $\mathbf{R}$ is a symmetric positive-definite matrix. Hence, the performance index is represented as $J = J_y + \rho\, J_u$.

The well known solution to the above control coupled output regulation problem given in [18] is now briefly reviewed. As discussed in [18], with the definition $\overline{\mathbf{u}}(\tau) =$ $\mathbf{u}(\tau) + \mathbf{R}^{-1}\mathbf{N}^T\mathbf{x}(\tau)$, the Eq. (6) becomes equivalent to

$$
\dot{\mathbf{x}} = \left(\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{N}^T\right)\mathbf{x} + \mathbf{B}\overline{\mathbf{u}} \tag{10}
$$

and the performance index in Eq. (9) is modified to

$$
\begin{aligned}
J_R &= \int_{t_0}^{\infty} [\left(\mathbf{u}(\tau) + \mathbf{R}^{-1}\mathbf{N}^T\mathbf{x}(\tau)\right)^T \mathbf{R}\left(\mathbf{u}(\tau) + \mathbf{R}^{-1}\mathbf{N}^T\mathbf{x}(\tau)\right) + \mathbf{x}^T(\tau)\left(\mathbf{Q} - \mathbf{N}\mathbf{R}^{-1}\mathbf{N}^T\right)\mathbf{x}(\tau)]\ \ d\tau \\
&= \int_{t_0}^{\infty} [\overline{\mathbf{u}}^T(\tau)\mathbf{R}\overline{\mathbf{u}}(\tau) + \mathbf{x}^T(\tau)\left(\mathbf{Q} - \mathbf{N}\mathbf{R}^{-1}\mathbf{N}^T\right)\mathbf{x}(\tau)]\ \ d\tau
\end{aligned} \tag{11}
$$

The optimal control law for $\overline{\mathbf{u}}$ is given by $\overline{\mathbf{u}}^* = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}\,\mathbf{x}(\tau)$

where $\mathbf{P}$ is the solution of the following Algebraic Riccati Equation (ARE)

$$
\overline{\mathbf{P}}\left(\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{N}^T\right) + \left(\mathbf{A}^T - \mathbf{N}\mathbf{R}^{-1}\mathbf{B}^T\right)\overline{\mathbf{P}} - \overline{\mathbf{P}}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\,\overline{\mathbf{P}} + \mathbf{Q} - \mathbf{N}\mathbf{R}^{-1}\mathbf{N}^T = 0 \tag{12}
$$

Therefore, the final optimal control law for $\mathbf{u}$ associated with Eq. (6) and Eq. (11) is given by

$$
\mathbf{u}^* = -\mathbf{R}^{-1}\left(\mathbf{B}^T\mathbf{P} + \mathbf{N}^T\right)\mathbf{x}(\tau) = -\mathbf{K}^*\mathbf{x}(\tau) \tag{13}
$$

$$
\mathbf{K}^* = \mathbf{R}^{-1}\left(\mathbf{B}^T\mathbf{P} + \mathbf{N}^T\right) \tag{14}
$$

In addition, the Close Loop system is asymptotically stable if

(a) The pair (**A**, **B**) is stabilizable;

(b) $\mathbf{R} = \mathbf{R}^T > 0$ and $\mathbf{Q} - \mathbf{N}\mathbf{R}^{-1}\mathbf{N}^T \geq 0$; and

(c) The pair ($\mathbf{Q} - \mathbf{N}\mathbf{R}^{-1}\mathbf{N}^T$, $\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{N}^T$) has no unobservable mode on the imaginary axis.

In our design process, we assume the pair (**A**, **B**) is controllable and pair (**A**, **C**) is observable.

Note that the weighting matrix for the cross term between the state and control variables $\mathbf{N}$ is involved in the ARE in Eq. (12) and in the optimal controller gain $\mathbf{K}^*$ in Eq. (14).

As mentioned before, in the LQR design for uncoupled (standard) output regulation problems, J is made zero only with an infinite control effort. But in control coupled output regulation, under some circumstances, it is possible for J to approach to zero or even identically become equal to zero with finite control effort.

Note that in the above optimal control solution, the scalar design variable ρ is implicit in the control weighting matrix **R**, which in turn determines the optimal control gain $\mathbf{K}^*$. The objective of our current investigation is to provide guidelines for selecting the best ρ for control coupled output regulation problem, in which it is sometimes possible to achieve zero output regulation with finite control effort. These proposed guidelines for selecting this scalar design variable ρ and the corresponding control effort very much depend on the nature or properties of a controller, which we label as 'Output Zeroing' controller $\mathbf{u}_{oz}$, which can be built, independent of the optimality issue.

In what follows, we present a thorough analysis of control coupled output regulation problem, with the help of the so called 'Output Zeroing' controller. Before discussing the role of this 'Output Zeroing' controller, it is important to notice few interesting features of the optimal controller for the control coupled output regulation problem. It is seen that the optimal control gain $\mathbf{K}^*$ has two terms, namely

$$\mathbf{K}^* = \mathbf{K}_P^* + \mathbf{K}_N^* \tag{15}$$

where $\mathbf{K}_P^* = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}$ and $\mathbf{K}_N^* = \mathbf{R}^{-1}\mathbf{N}^T$.

Out of these two terms, it may be seen that the $\mathbf{K}_p^*$ gain contributes more to achieving the stability of the closed loop system (since it is associated with the Riccati matrix) whereas the $\mathbf{K}_N^*$ gain contributes more towards achieving the objective of keeping y small i.e. J_y small (or even identically equal to zero in some situations, as we observe later). In other words, it can be concluded that in the uncoupled (standard) LQR problem, the entire control gain works towards stabilization while simultaneously trying to minimize J_y whereas in the coupled case, these two tasks, namely stabilization and minimization are accomplished in somewhat separate ways. While this conceptual observation is given more mathematical interpretation in the later sections, it is still useful to keep this conceptual idea in mind when we draw conclusions about the differences between the uncoupled and coupled cases in LQR framework. The relative contributions of these two gain components is essentially dictated by the nature (or properties) of the 'Output Zeroing' controller whose role is elaborated and discussed in the next section.

2.1.6.1 Output Zeroing Controller and its Relationship to the Optimal Controller when Matrix D is Square and Invertible

In this section, we discuss the so called 'Output Zeroing' controller and its relationship to the 'Optimal' controller. After all, the very purpose of addressing an optimal control formulation is to achieve, ideally, zero output. In the uncoupled output case, there is no option but to resort to the optimal control formulation, where stability of the closed loop system and output (state) regulation are simultaneously achieved by the

optimal controller. But in the coupled output case, there is indeed another option available, albeit with a caveat, but the point is that there is an additional option. It could be such an option that it can force the optimal controller towards this option. In this paper, we focus our attention to the case of matrix **D** being square and invertible. Then it is easy to observe that when $\mathbf{u}(\tau) = -\mathbf{D}^{-1}\mathbf{C}\mathbf{x}(\tau)$ is applied, the output is simply zero because in $\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} = (\mathbf{C} - \mathbf{D}\mathbf{D}^{-1}\mathbf{C})\mathbf{x} = \underline{0}$. We call this controller as 'Output Zeroing' Controller $\mathbf{u}_{oz}$. In addition, let us assume that the resulting closed loop system, which is $\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})\mathbf{x} = (\mathbf{A} - \mathbf{B}\mathbf{K}_{oz})\mathbf{x} = \mathbf{A}_{CLoz}\mathbf{x}$, is stable. Then, in this case, it is clear that the original objective of achieving zero output is accomplished, with a specific control effort corresponding to $\mathbf{u}_{oz}$. Note that if we evaluate the performance index $\overline{J}_y$ with the 'Output Zeroing' controller alone, we observe that $\mathbf{u}_{oz} = -\left(\mathbf{D}^{-1}\mathbf{C}\right)\mathbf{x}(\tau)$, and the performance index components are

$$\overline{J}_y = \int_{t_0}^{\infty} \Big[\mathbf{x}^T(\tau)(\mathbf{C}^T\overline{\mathbf{Q}}\mathbf{C})\mathbf{x}(\tau) + 2\mathbf{x}^T(\tau)\mathbf{C}^T\overline{\mathbf{Q}}\mathbf{D}\mathbf{u}_{oz}(\tau) + \mathbf{u}_{oz}{}^T(\tau)(\mathbf{D}^T\overline{\mathbf{Q}}\mathbf{D})\mathbf{u}_{oz}(\tau) \Big] d\tau$$

When the state regulation cost ($\overline{J}_x$), the control effort ($\overline{J}_u$) and the coupling effect ($\overline{J}_c$) are evaluated with $\mathbf{u}_{oz} = -\left(\mathbf{D}^{-1}\mathbf{C}\right)\mathbf{x}(\tau)$, they are given by:

$$\overline{J}_x = \int_{t_0}^{\infty} \mathbf{x}^T(\tau)\left(\mathbf{C}^T\overline{\mathbf{Q}}\mathbf{C}\right)\mathbf{x}(\tau)d\tau;$$

$$\overline{J}_u = \int_{t_0}^{\infty} \mathbf{u}_{oz}{}^T(\tau)(\mathbf{D}^T\overline{\mathbf{Q}}\mathbf{D})\mathbf{u}_{oz}(\tau)d\tau = \int_{t_0}^{\infty} \mathbf{x}^T(\tau)\left(\mathbf{C}^T\overline{\mathbf{Q}}\mathbf{C}\right)\mathbf{x}(\tau)d\tau;$$

$$\overline{J}_c = 2\int_{t_0}^{\infty} \mathbf{x}^T(\tau)\mathbf{C}^T\overline{\mathbf{Q}}\mathbf{D}\mathbf{u}_{oz}(\tau)\, d\tau == -2\int_{t_0}^{\infty} \mathbf{x}^T(\tau)\left(\mathbf{C}^T\overline{\mathbf{Q}}\mathbf{C}\right)\mathbf{x}(\tau)\, d\tau.$$

Note that $\overline{J}_x = \overline{J}_u$ and $\overline{J}_c = -2\overline{J}_x$. Thus, $\overline{J}_y = \overline{J}_x + \overline{J}_u + \overline{J}_c = 0$. That is, with the 'Output Zeroing' controller, assuming the closed loop system $\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})\mathbf{x} = (\mathbf{A} - \mathbf{B}\mathbf{K}_{oz})\mathbf{x} = \mathbf{A}_{CLoz}\mathbf{x}$ to be stable, we have $\overline{J}_x = \overline{J}_u$ and $\overline{J}_c = -2\overline{J}_x$. This implies that when $\overline{J}_c$ is negative, it is equivalent to the statement that the control and state variables are working together or assisting each other and in that case, it is possible to achieve zero output regulation with finite control effort.

In what follows, we consider two situations for square matrix **D** case and discuss the relative trade offs between J_y and J_u and provide concrete guidelines for the selection of the design variable ρ.

Case I: D is a square matrix (k = m) and Invertible, and $\mathbf{A}_{CLoz}$ is stable (and thus $\overline{\mathbf{Q}} = \mathbf{I}$):

For this case, **y** is identically zero using the 'Output Zeroing' Controller and since $\mathbf{A}_{CLoz}$ is stable, there is no need even to provide any weighting to the output vector. That

is the reason $\overline{\mathbf{Q}}$ is taken as Identity matrix. If we now carryout the optimal control gain determination, the forcing function matrix in the Riccati equation is an identically zero matrix and thus $\mathbf{P}=0$ becomes a solution to the Riccati equation and hence the optimal control gain simply contains only the minimization component $\mathbf{K}_N^*$, with the stabilization component $\mathbf{K}_P^*$ becoming zero. In other words, we can safely take the design variable ρ to be identically zero.

When $\rho = 0$ and $\overline{\mathbf{Q}} = \mathbf{I}$, $\mathbf{K}_P^* = \underline{0}$ and

$$\mathbf{K}_N^* = \mathbf{R}^{-1}\mathbf{N}^T = \left(\mathbf{D}^T\mathbf{D}\right)^{-1}\left(\mathbf{D}^T\mathbf{C}\right) = \mathbf{D}^{-1}\mathbf{C}\mathbf{x}(t) = \mathbf{K}_{oz}(t) .$$

Note that the control effort J_u corresponding to this optimal control gain is the largest control effort needed to make the output $\mathbf{y}=0$ and it is finite. The important point to note here is that for this case, zero output regulation can be achieved with finite control effort which is definitely not the case for the standard, uncoupled case. When $\mathbf{A}_{CLoz}$ is stable, it implies that minimization task and stabilization are achieved simultaneously by using 'Output Zeroing' control such that minimizing performance output(s) and state(s) are coincident.

<u>Guideline for selection of ρ</u>: Thus, for this very special case, the 'optimal' control gain through the LQR framework is simply obtained by taking $\rho = 0$. For any other positive ρ value, the control effort with that gain will be lower with a corresponding increase in the output regulation cost J_y.

Case II: D is a square matrix (k = m) and Invertible, and $\mathbf{A}_{CLoz}$ is unstable (and thus $\overline{\mathbf{Q}}$ is any fixed symmetric positive definite matrix):

For this case, **y** is still identically zero using the 'Output Zeroing' controller. However, in this case, because the matrix $\mathbf{A}_{CLoz}$ is not stable, we need to carryout the optimal control gain determination by varying ρ in a nonzero positive range satisfying the assumptions on the forcing function matrix in the Riccati equation, namely that, $\mathbf{Q}-\mathbf{N}\mathbf{R}^{-1}\mathbf{N}^T \geq 0$. We thus we need both the output weighting matrix $\overline{\mathbf{Q}}$ as well as the control weighting matrix $\rho\overline{\mathbf{R}}$ to ensure the satisfaction of this condition. The optimal control gain now contains both the stabilization component $\mathbf{K}_P^*$ as well as the minimization component $\mathbf{K}_N^*$. It is easy to observe that most of the effort would go to stabilization and very little effort going to minimization task. The output regulation cost J_y then approaches $J_{y\min}$ as control effort J_u keeps increasing.

<u>Guideline for selection of ρ</u>: In this situation, the selection of ρ is such that it can be picked to be a value away from zero in an increasing fashion and the final desired value can be selected based on the minimum control effort J_u needed for achieving the inevitable $J_{y\min}$. The selection of ρ thus involves both J_u and J_y considerations.

In control coupled regulation problem, the value of $J_{y\min}$ which can be close to zero or away from zero is determined by the nature of system (output minimization and

stability) and the interaction between the states and control inputs through the matrices **A**, **B**, **C** and **D**.

2.1.7 Application to the 3DOF Vehicle Dynamics Problem

In this section, we apply the above LQR extension procedure to the 3DOF vehicle dynamics problem discussed before and investigate the performance of the resulting controllers under a double lane change maneuver as a disturbance. In the simulations, a yaw torque as a disturbance acting on the vehicle during the double lane change is considered to illustrate the performance of active steering controller design on the rollover prevention objective. An unexpected disturbance torque can lead to dangerous driving situations because of the overreactions. Then, the excessive steering form a driver can cause the vehicle to rollover. An automatic feedback control system designed using the method proposed in this paper can react faster and more precisely than a human driver without the excessive steering. A periodic disturbance torque is not typical, so a step disturbance is adopted in the simulations. The peak value of the yaw torque disturbance is 14K Nm and it is taken at 1-second intervals (form 1sec to 2sec). The amount of yaw torque disturbance is assigned a sufficiently large value to raise the rollover risk. A constant forward velocity (v_x) of 100km/hr is assumed in all the simulations.

The vehicle model with active steering control and active roll moment control are evaluated separately and their performances compared. The following simulation is mainly used to show the difference between Output Zeroing control and standard LQR control and thus no actuator saturation and actuator dynamics are considered. Also, false activation is not an issue here since the external yaw torque disturbance is not induced from steering. That is, the controller is simulated to prevent the vehicle from rollover under assumed external yaw torque disturbance such as wind gust rather than by steering induced lateral acceleration.

Active steering control:

(1) For this case, $\mathbf{A}_{CLoz}$ is stable, so it belongs to case I.

(2) The optimal control is 'Output Zeroing' control where $\mathbf{K}_{oz}(t) = \mathbf{D}^{-1}\mathbf{C}\mathbf{x}(t)$; $J_y = 0$; and control gain is

$$\mathbf{K}_{oz} = [1.1788 \quad -0.0891 \quad -0.0165 \quad 0.0648].$$

Active roll moment control:

(1) For this case, $\mathbf{A}_{CLoz}$ is unstable, so it belongs to case II.

(2) $\rho = 8.38\times10^{-11}$ is selected such that the control input is reasonable and control gain is

$$\mathbf{K} = [-52974 \quad 2940 \quad 11124 \quad 47.852].$$

Note that the gain values for roll moment control are much larger compared to the gain values for active steering control. This is reasonable because the steering angles are required to be small in our linear design problem. Also the gain values are large in the roll moment control because these roll moments are perceived to be generated by either suspension systems or anti-roll bars with heavy hardware.

The simulation results of rollover index for the vehicle with active steering control, active roll control and without control are shown in Fig. 2. The corresponding control inputs (steering angle and roll moment) are shown in Fig.3.

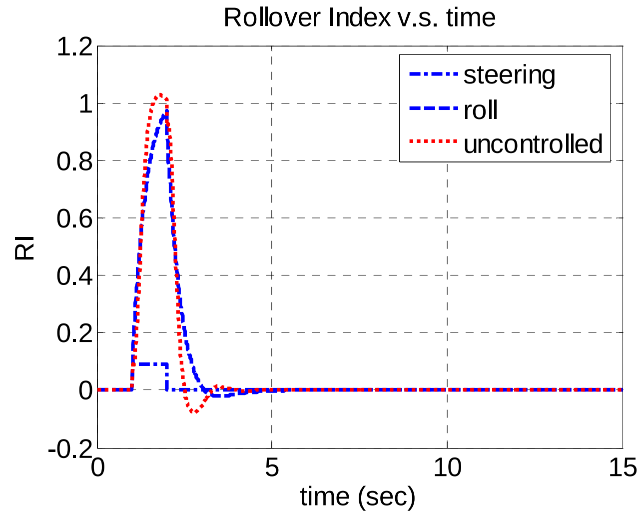

FIGURE 2. ROLLOVER INDICES FOR THE VEHICLE WITH ACTIVE STEERING CONTROL, ACTIVE ROLL CONTROL AND WOTHOUT CONTROL RESPECTIVELY

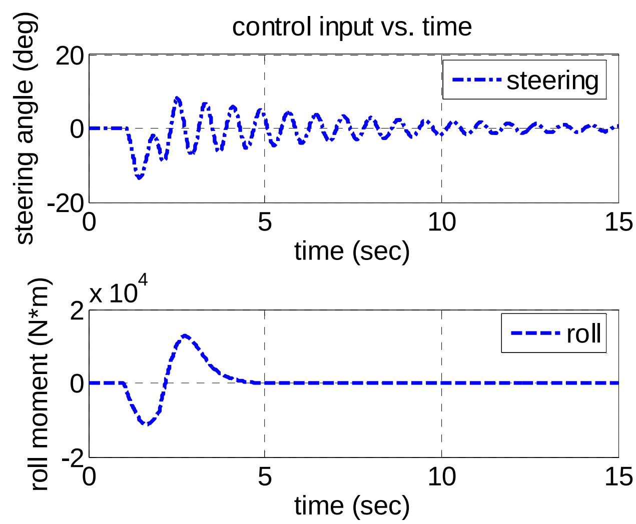

FIGURE 3. CONTROL INPUTS FOR THE VEHICLE WITH ACTIVE STEERING CONTROL AND ACTIVE ROLL CONTROL

Note that the uncontrolled vehicle rolls over and controlled vehicles do not. It is interesting to observe that the steering control which is basically the Output Zeroing controller naturally makes RI identically equal to zero in a short time. The roll control which has optimal control component for stabilization naturally has higher RI in the transient phase. The active steering control based on ‘Output Zeroing’ control improves the rollover significantly under the disturbance torque. However, compared to roll

moment control based on optimal control, this controller takes longer to settle down because in this controller, the gain component for output minimization ($\mathbf{K}_N^*$) is more active than the stabilization component ($\mathbf{K}_P^*$).

2.1.8 Conclusions

In this paper, a novel control system analysis and design technique is presented by extending the popular Linear Quadratic Regulator (LQR) control design method to the 'control coupled output regulation' problem. In control coupled output regulation of linear state space systems, the performance index being minimized in the LQR framework, results in a coupled term in state and control variables. While there are guidelines to select weightings on the state or control variables in the uncoupled regulation problem, there are no such guidelines available for the coupled case. In this paper, we present considerable insight into the major differences in the optimal control problem formulation for the uncoupled and coupled cases and provide guidelines for weight selections by taking into consideration the interaction (or coupling) between state and control terms for square and invertible matrix **D** case and using the property of 'Output Zeroing' controller. The relationship and relative trade offs between output zeroing controller and the optimal controller are presented. The proposed analysis and design method is illustrated with rollover prevention control for a multi-body Ground Vehicle using the active steering control and active roll control respectively.

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2.2 Active Roll Control for Rollover Prevention of Heavy Articulated Vehicles with Multiple-Rollover-Index Minimization

2.2.1 Abstract

This paper presents the application of a nominal control design algorithm for rollover prevention of heavy articulated vehicles with active anti-roll bar control. This proposed methodology is based on an extension of Linear Quadratic Regulator (LQR) control for ‘state derivative induced (control coupled) output regulation’ problems. For heavy

articulated vehicles with multiple axles, a performance index with Multiple Rollover Indices (MRI) is proposed. The proposed methodology allows us to compare the usefulness of various control configurations (i.e. actuators at different axles of the vehicle) based on the interaction of this control configuration with vehicle dynamics. Application of this methodology to a specific heavy articulated vehicle with tractor semi-trailer shows that a single active anti-roll bar system at the trailer unit gives better performance than multiple-axle actuators at tractor and trailer together with the double lane change maneuver as the external disturbance.

2.2.2 Introduction

Rollover prevention is a fundamental and significant issue for vehicle dynamics and has been a topic of considerable research for a long time [1-14]. Rollovers are divided into two broad categories: tripped and un-tripped. A tripped rollover commonly occurs when a vehicle skids and digs its tires into soft soil or hits a tripping mechanism such as a curb or guardrail with a sufficiently large lateral velocity. Maneuver-induced un-tripped rollover can occur during typical driving situations and poses a real threat for the vehicles with an elevated center of gravity. Examples are excessive speed during cornering, obstacle avoidance and severe lane change maneuvers, where rollover occurs as a result of the lateral wheel forces induced during these maneuvers. Furthermore, vehicle rollover can also occur during external disturbances like side-wind, steering excitation, etc. Thus, passenger vehicles with a high center of gravity such as light trucks (vans, pickups, and SUVs (Sport Utility Vehicles)) are more prone to rollover accidents. Moreover, the heavy commercial vehicles with narrow track width are often involved in rollover accidents. Also, rollover prevention is of high importance for military vehicles, which operate in severe operational environments and maneuvers. Rollover prevention is a critical safety issue but there is still no safety performance standard available. In addition, there are no well-recognized rollover protection standards or design guidelines.

Typically, a driver does not have any indication before a rollover happens and many rollover situations cannot be prevented by driver actions alone, even when they are correctly warned. Additional assistance from active anti-rollover control can mitigate the deficiency in human capability. Hence, rollover prevention systems are classified into two stages: detection of the possibility of a rollover, and development of a mitigation control algorithm. Thus, research on rollover prevention systems has mainly focused on two areas: rollover detection systems and anti-rollover control systems. In this paper, the emphasis is on developing an efficient control configuration for anti-rollover control systems.

After the risk of rollover is detected, effective anti-rollover control systems are crucial to prevent vehicles from rollover or help vehicles to recover from rollover. In recent years with the development of advanced control technology and cost reduction of electronic and control equipment, active control has been widely used in the automotive industry in the design of anti-rollover control systems. There are four major active rollover control applications based on the actuation schemes: (1) Active anti-roll-bar systems; (2) Active suspension systems (3) Anti-roll braking systems; and (4) Active steering systems. Active anti-roll-bar and active suspension systems directly control the vehicle roll motion. Anti-roll braking systems and active steering systems reduce vehicle oversteer and control

vehicle yaw moment. An active anti-roll-bar hydraulically determines the variation of the equivalent stiffness of the anti-roll-bars. The vehicle load distribution is influenced by the roll-bar stiffness distribution such that the roll angle and roll moment are improved by an active anti-roll-bar. The use of an anti-roll-bar system to improve vehicle roll stability and reduce the rollover has been proposed and developed, especially for heavy road vehicles [8-12]. Active suspension systems use electrohydraulic equipment to generate controlled vertical forces to react to rollover moments and are used to gain improvements in both roll and ride performance [3,4]. Both anti-rollover braking system and active steering system essentially control yaw moment to reduce rollover risk indirectly due to the coupling between roll, lateral, and yaw dynamics. An anti-rollover braking system controls the front brakes to reduce the cornering capability of the front tires, which causes the vehicle to turn less sharply and reduces its speed to prevent the rollover [1,2,5,13]. Active steering systems control the steering input directly to reduce the rollover risk by reducing or reversing the steering angle to reduce or reverse the unstable roll maneuver [6,7,14,19]. In addition, combinations of these different techniques are also considered [3,7]. In this paper, we focus our attention on active anti-roll-bar control system and present an effective control design methodology for a 9-Degrees Of Freedom (DOF) tractor-semi-trailer. It may be noted that active anti-roll-bar system is a common and desirable controller for heavy vehicles [10]. In this paper, we use the mathematical model for vehicle dynamics developed in [10].

The paper is organized as follows: in section 2.2.3, vehicle dynamics modeling for heavy articulated tractor-semi-trailer is described. In section 2.2.4, a new unified rollover index is introduced and then extended to Multiple Rollover Indices (MRI) for heavy vehicles with multiple axles. In section 2.2.5, a rollover performance index is proposed to be used as an output in the optimal control formulation of Linear Quadratic Regulator (LQR) framework. A brief introduction to state derivative induced (control coupled) output regulation is presented in section 2.2.6. In addition, a controller selection algorithm based on state derivative induced (control coupled) output regulation is presented. In section 2.2.7, the controller selection algorithm developed in section 2.2.6.1 is applied to the rollover prevention problem of the 9-DOF heavy articulated tractor-semi-trailer vehicle. Three control configurations, namely i) anti-roll-bars on all three axles (tractor steering axle, tractor drive axle and trailer unit) ii) anti-roll-bars on two axles (tractor drive axle and trailer unit) iii) anti-roll-bars on only one axle (trailer unit) are compared and it is shown that using the proposed methodology, for the specific vehicle model considered, a single anti-roll-bar at the trailer can give better rollover performance than multiple actuators on all axles. In section 2.7.8, a simulation for double lane change maneuver is conducted and results are discussed. Finally, section 2.7.9 offers some concluding remarks.

2.2.3 Vehicle Dynamics Modeling for Heavy Articulated Tractor Semi-Trailer Vehicle

In order to develop the most complete model of vehicle roll behavior and examine roll response associated with specific maneuvering conditions, it is necessary to consider the vehicle model combining motions both in the yaw and roll planes. That is because lateral, yaw and roll dynamics are all coupled. Therefore, in this paper, a detailed handing model which includes lateral, yaw, and roll dynamics is considered as a rollover model. In these

rollover models, external inputs like road profile, steering angle from a driver, and wind gust are treated as disturbances. A higher order non-linear vehicle model is linearized locally around an operating point by assuming small angle approximation and by neglecting higher order dynamic terms and the resulting linearized model is used for controller design. In these models, vehicle forward velocity is taken as a constant varying parameter. Towards this objective, the mathematical model of the tractor-semi-trailer in [10] which includes frame flexibility also is considered. As discussed in [10], the frame flexibility is to capture the influence of compliance on the distribution of roll moments between axles. The sprung mass of tractor is split, typically in proportion with the axle weights, into front and rear sections, each with appropriate inertial properties. A kinematic constraint between adjacent vehicle units, which are a tractor unit and a semi-trailer, is included in the equations of motion. Tyre parameters are also included in this tractor semi-trailer model. The tractor-semi-trailer model considered has 9 DOF: six degrees of freedom of the tractor unit (yaw, side-slip, front and rear sprung mass roll angle, steer axle roll angle and drive axle roll angle), plus the articulation angle between the tractor and trailer, the roll angle of the sprung mass of the trailer, and the roll angle of the trailer axle group. The nonlinear model is linearized locally around an operating point. The equations of motion for the linear tractor semi-trailer vehicle model are shown in [10].

The thirteen states of the system are roll angle of front and rear sprung mass for tractor ($\phi_{f,1}$ and $\phi_{r,1}$), roll rate of front and rear sprung mass for tractor ($\dot{\phi}_{f,1}$ and $\dot{\phi}_{r,1}$), sideslip angle for tractor (β_1), yaw rate for tractor ($\dot{\psi}_1$), roll angle of front and rear unsprung mass for tractor ($\phi_{t,f,1}$ and $\phi_{t,r,1}$), roll angle of sprung mass for trailer (ϕ_2), roll rate of sprung mass for trailer ($\dot{\phi}_2$), sideslip angle for trailer (β_2), yaw rate for trailer ($\dot{\psi}_2$), and roll angle of unsprung mass for trailer ($\phi_{t,r,2}$). $u_{f,1}$ is the active roll torque at the tractor steering axle, $u_{r,1}$ is the active roll torque at the tractor drive axle, $u_{r,2}$ is the active roll torque at the trailer unit. δ is the front wheel steering angle from driver. Subscript 1 denotes tractor, subscript 2 denotes trailer, subscript *f* denotes front and subscript *r* denotes rear. All parameters of interest and the numerical values of the parameters of the model are taken from [10]. The equations of motion are written in state space form as follows:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{B}_w\mathbf{w} \tag{1}$$

where x is the state vector, u is the active roll torque vector and $\mathbf{w}$ is the steering angle from a driver, which is treated as a disturbance.

2.2.3.1 Rollover Index (RI) for A single Unit Vehicle

Rollover Index is an important metric in vehicle safety assessment. A variety of rollover indices have been introduced in the literature [5]. In rollover detection systems, the concept of a rollover index is used to determine the threshold for rollover. Various rollover thresholds are derived from different factors which influence rollover such as the position of a vehicle's center of gravity (CG), the energy of rollover, and vertical tire forces (Lateral Transfer Ratio (LTR)). Moreover, a method to identify real-time predictive Lateral Transfer Ratio (PLTR) is introduced in [16]. In this paper, we develop a 'unified

rollover index' using the concept of LTR described in [5,6] by taking the torque balance for an unsprung mass about the zero-level center mentioned in [17]. Note that once wheel lift occurs, anti-rollover control is very difficult. Therefore, the concept of LTR is selected in this research and incorporated in the performance index of LQR framework.

Rollover index is defined as the load difference (i.e., vertical force) between the inside and outside wheels of the vehicle, normalized by the total load. That is,

$$\begin{aligned}\mathrm{RI} &= \frac{\text{Load on Outside Tires} - \text{Load on Inside Tires}}{\text{Total Load}}\\ &= \frac{F_{z,o} - F_{z,i}}{F_{z,o} + F_{z,i}} = \frac{F_{z,o} - F_{z,i}}{mg}\end{aligned}$$

where g is gravity, m is mass of the total vehicle and $m = m_s + m_u$ (m_s is sprung mass and m_u is unsprung mass). With this definition of the index, the vehicle is considered 'rolled over' when the Rollover Index (RI) is equal to 1 or -1. In other words, the vehicle does not roll over as long as $|\mathrm{RI}| < 1$. For $F_{z,o} = F_{z,i}$ (i.e., RI = 0), the vehicle drives straight on a horizontal road. When $F_{z,o} = 0$ (i.e., RI = -1), the outside wheels lift off the road. When $F_{z,i} = 0$ (i.e., RI = 1), the inside wheels lift off the road.

In [13,14], the authors argued that the rollover estimation in [6] is not sufficient to detect the transient phase of rollover due to the fact that it is derived ignoring roll dynamics. Furthermore, in [13,14] the rollover index is given by $\mathrm{RI} = -\frac{2(k\phi + c\dot{\phi})}{mgT}$, where T is track width, ϕ and $\dot{\phi}$ is roll angle and roll rate of body 2 relative to body 1 respectively and k, c denote the total torsional spring stiffness and torsional damper coefficients of the suspension respectively. However, even this is also not sufficient to estimate the rollover since the lateral dynamics is ignored in the formula. The lateral acceleration is a critical factor in rollover. Therefore, in what follows, we present a new rollover index which includes the lateral and roll dynamics simultaneously.

A schematic of the vehicle rollover model is shown in Fig. 1. Body 1 is unsprung mass ($m_u = m_1$) and CG1 is center of gravity of body 1. Body 2 is the sprung mass ($m_s = m_2$) and CG2 is center of gravity of body 2. The unsprung mass is assumed to be insignificant and assumed to roll about a horizontal roll axis, which is along the centerline of the unsprung mass and at the ground level. Hence, it is reasonable to neglect the unsprung mass inertial contribution. The effective linear torques exerted by the suspension system about the roll center are defined as $T_{\text{spring}} = k\phi$ and $T_{\text{damper}} = c\dot{\phi}$, where ϕ is roll angle and $\dot{\phi}$ is roll rate of body 2 relative to body 1 and k, c denotes the total torsional spring stiffness and torsional damper coefficients of the suspension respectively. The unified rollover index is derived from the torque balance of the unsprung mass about the zero-level center, which is the point S in the Fig. 1. The result of the torque balance is shown in Eq. (2).

$$\begin{cases} (-F_{z,o}+F_{z,i})\dfrac{T}{2}+F_y h_R+k\phi+c\dot{\phi}=0 \\ F_y=m_2 a_{y,2}=m_2(\dot{v}_y+v_x r-h\ddot{\phi}) \end{cases} \tag{2}$$

$$\begin{aligned} \Rightarrow F_{z,o}-F_{z,i} &= \frac{2}{T}\left(m_2 a_{y,2} h_R+k\phi+c\dot{\phi}\right) \\ &= \frac{2}{T}\left[m_2(\dot{v}_y+v_x r-h\ddot{\phi})h_R+k\phi+c\dot{\phi}\right] \end{aligned} \tag{3}$$

where F_y is lateral force, $a_{y,2}$ is the lateral acceleration of the CG2, h_R is distance from the roll axis to the ground, h is distance from the CG2 to the roll axis, T is track width, r is yaw rate of body 1, v_x is longitudinal velocity of body 1, $\dot{v}_y$ is the lateral acceleration of body 1, and $\ddot{\phi}$ is roll acceleration of body 2 relative to body 1. In a straight driving condition with constant forward velocity v_x, the lateral acceleration of body 2 is given by $a_{y,2}=\dot{v}_y+vr-h\ddot{\phi}=\dot{v}_y+v_x r-h\ddot{\phi}$.

Hence, the rollover index can finally be expressed as

$$\mathrm{RI}=\frac{F_{z,o}-F_{z,i}}{F_{z,o}+F_{z,i}}=\frac{F_{z,o}-F_{z,i}}{mg}=\frac{2[m_2(\dot{v}_y+v_x r-h\ddot{\phi})h_R+k\phi+c\dot{\phi}]}{mgT} \tag{4}$$

Since both roll and lateral dynamics can affect rolling moment, the unified rollover index presented in the above Eq. (4), which includes the roll and lateral dynamics, is more appropriate and effective in detecting rollover.

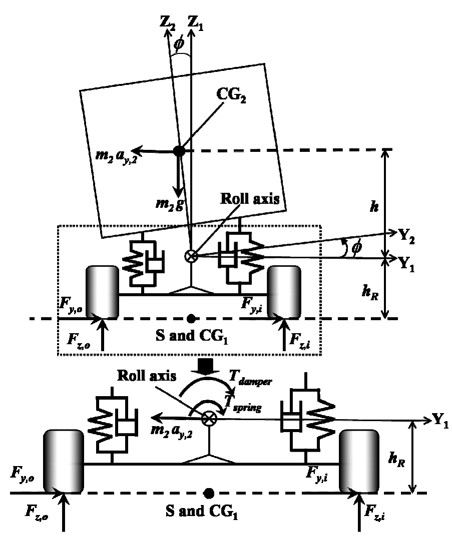


FIGURE 1. VEHICLE ROLLOVER MODEL

2.2.3.2 Multiple Rollover Indices for Heavy Articulated Vehicles

The vehicle frame is usually assumed to be a rigid body for a small, single-unit

vehicle. However, torsional compliance of the vehicle frame has influence on the distribution of roll moments between axle groups for articulated heavy vehicles. Winkler et al. noted that the torsional compliance of the vehicle's structure frame can contribute to the rollover [15]. That is, highly compliant (flat bed) trailers may roll over at the trailer unit without lifting driver-axle tires. Since every axle can roll over independently in a multiple axle vehicle, we propose to define a rollover index, such as one described in Eq. (4), for each axle separately.

For this tractor semi-trailer, there are three rollover indices defined. One index is for tractor steering axle (RI_1), another index is for tractor drive axle (RI_2), and the other index is for the group of trailer axles (RI_3), where three trailer axles are lumped together as a group. From the definition of the rollover for tractor semi-trailers, this tractor semi-trailer is considered 'rolled over' when both the drive axle and trailer unit have lifted off the road [10].

2.2.4 Rollover Index as an Output (Controlled Variable)

It is important to realize that the rollover index given in Eq. (4) is strictly a function of the state variables and their derivatives, where the state variables are given by $v_y, \phi, \dot{\phi}$ and r. Since rollover prevention is our primary control objective, we propose to express the Rollover Index (RI) itself as an output variable (y) in the state space form of the vehicle dynamics. Recall that state space form of the vehicle dynamics is described in section 2.2.3 and in the absence of the external disturbances is given by

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \tag{5}$$

When rollover index itself is expressed as an output variable (y), it can be seen that y is linear function of x and $\dot{\mathbf{x}}$.

2.2.5 State Derivative Induced Output Regulation Problem

Our primary objective now is to design a controller to prevent rollover. This can be accomplished by expressing the rollover index as an output variable and then regulating the output y. One popular design method for regulating the output is the LQR method [18]. Since in our current rollover prevention problem, the output y is a linear function of the state x and its derivative $\dot{\mathbf{x}}$, it is possible to consider the state derivative $\dot{\mathbf{x}}$ to be generated from a state space model $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$ where we can model the Bu term to comprise various actuators. Thus Bu can be $[b_1 \quad b_2 \quad b_3]\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ (with three actuators), $[b_1 \quad b_2]\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ (with two actuators), or $b_1 u_1$ (with only one actuator). Note that accordingly, the output y (i.e. the rollover index) can also then be expressed as $\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$ where again the Du term can be can be $[d_1 \quad d_2 \quad d_3]\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ (with three actuators), $[d_1 \quad d_2]\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ (with two actuators), or $d_1 u_1$ (with only one actuator).

In other words, in this representation, the rollover index is a direct function of the

performance of the various actuators and is thus highly influenced and determined by the actions of these different control configurations. Thus, this idea of representing rollover index as an output of a linear state space system and then incorporating this index into the performance index in the LQR framework allows us to compare the efficacy of these different control configurations in the rollover prevention problem. Since in this situation, the output is expressed as $\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$, this regulation problem in LQR framework is labeled as Control Coupled Output Regulation problem. In this paper, we apply the extension of LQR method to the control couple output regulation problem, not only to design an appropriate controller for rollover prevention but also to compare these different control configurations in their efficacy in preventing rollover.

2.2.5.1 Multiple Rollover Indices (MRI) as Output Components and a Rollover Performance Index

In [19], an approach in which the single rollover index in $\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$ form is expressed as a performance index is discussed. In this paper, we consider all the three rollover indices RI_1, RI_2 and RI_3 as components of the output vector, resulting in the 3×1 output vector (y). Therefore, the controller for preventing three axles from rollover simultaneously is efficiently designed based on this representation of output equation without which analyzing the complicated roll-over coupling between all axles would be very difficult. We now consider three ‘control configurations’ involving combinations of active roll bars at different axles described below and compare their effectiveness in reducing rollover using a novel control design extension to LQR procedure for control coupled output regulation. For control configuration 1, where all axles on vehicle have active roll control. For control configuration 2, where the tractor drive axle and the trailer unit have active roll control. For control configuration 3, where only the axles on the trailer unit have active roll control.

The state equation with control configuration 1 is given by

$$\dot{\mathbf{x}}_{13\times 1} = \mathbf{A}_{13\times 13}\mathbf{x}_{13\times 1} + \mathbf{B}_{13\times 3}\begin{bmatrix} u_{f,1} \\ u_{r,1} \\ u_{r,2} \end{bmatrix}$$

and the output equation with control configuration 1 is given by

$$\mathbf{y}_{3\times 1} = \begin{bmatrix} \mathrm{RI1} \\ \mathrm{RI2} \\ \mathrm{RI3} \end{bmatrix} = \mathbf{C}_{3\times 13}\mathbf{x}_{13\times 1} + \mathbf{D}_{3\times 3}\begin{bmatrix} u_{f,1} \\ u_{r,1} \\ u_{r,2} \end{bmatrix}$$

where $\mathbf{x}$ is state vector with 13 states shown in section 2.2.3, and the elements in matrix C, and D for this case are shown in Appendix I. The output equation for control configuration 2 and 3 can be represented similarly. The output equations for these three control configurations can be incorporated into the performance index of LQR framework and the procedure is in [19].

2.2.6 State Derivative Induced (Control Coupled Output) Regulation Problem

Note that ‘State derivative induced output regulation’ problem is a special case of

control coupled output regulation problem.

In the LQR design based on rollover performance index, minimizing the rollover performance index is equivalent to minimizing the modulus of rollover index using an optimal controller. The performance index in LQR design for the control coupled output regulation problem is

$$J = \int_{t_0}^{\infty} \left[\mathbf{y}^T(\tau)\overline{\mathbf{Q}}\mathbf{y}(\tau) + \rho\, \mathbf{u}^T \overline{\mathbf{R}}\mathbf{u}\right] d\tau = J_y + \rho\, J_u$$

$$= \int_{t_0}^{\infty} \left[\mathbf{x}^T(\tau)\mathbf{Q}\mathbf{x}(\tau) + 2\mathbf{x}^T(\tau)\mathbf{N}\mathbf{u}(\tau) + \mathbf{u}^T(\tau)\mathbf{R}\mathbf{u}(\tau)\right] d\tau \qquad (6)$$

where $\mathbf{Q} = \mathbf{C}^T\overline{\mathbf{Q}}\mathbf{C}$; $\mathbf{N} = \mathbf{C}^T\overline{\mathbf{Q}}\mathbf{D}$; $\mathbf{R} = \mathbf{D}^T\overline{\mathbf{Q}}\mathbf{D} + \rho\overline{\mathbf{R}}$. Note that the weighting matrices $\overline{\mathbf{Q}}$ and $\overline{\mathbf{R}}$ are symmetric positive definite. The weighting matrix $\mathbf{Q}$ is a symmetric positive semi-definite and the weighting matrix $\mathbf{R}$ is a symmetric positive-definite matrix. J_y is performance output cost, J_u is control effort. In our design process, we assume the pair (**A**, **B**) is controllable and the pair (**A**, **C**) is observable.

Note that once the LQR design procedure for the control coupled output regulation problem is applied, J_y is never identically zero, but depending on the control effort, achieves some minimum value, denoted by $J_{y\min}$. The procedure involves plotting J_y vs J_u and then noting the value of $J_{y\min}$. It may be noted that this value of $J_{y\min}$ very much depends on the nature of system (output minimization and stability) and the interaction between the states and control inputs through the matrices A, B, C and D.

2.2.6.1 Controller Selection Algorithm

In our proposed LQR methodology, we assume full state feedback control, so all states are assumed to be measured or estimated. The objective is to decide which control configurations have a desirable effect on the output being regulated and provide a criterion for selecting the appropriate control configuration.

The value of $J_{y\min}$ reflects the performance of controller since the controller design is based on the nature of the system and (output minimization and stability) and the interaction between various variables. Hence, the value of $J_{y\min}$ can be used to offer a selection criterion for different control configurations in ‘state derivative induced (control coupled) output regulation’ problems.

Criterion: A closed-loop system that has smaller $J_{y\min}$ means that that particular control configuration can reduce the output significantly (assuming the corresponding control effort can be generated). So we use $J_{y\min}$ as a comparison index for assessing the efficacy of the various control configurations considered in the solution procedure.

2.2.7 Evaluation of the Different Control Configurations

In this section, the three control configurations, which are designed for the 9-DOF

tractor semi-trailer model with active anti-roll-bar control and shown in section 2.2.5.1 are evaluated. In order to apply criterion in section 2.2.6.1 to evaluate the LQR design with control coupled output regulation, $J_{y\min}$ is obtained by varying the weight ρ with a fixed $\overline{\mathbf{Q}} = \mathbf{I}$ and $\overline{\mathbf{R}} = \mathbf{I}$.

Control Configuration 1: $J_{y\min} = 7.849 \times 10^{-15}$

Control Configuration 2: $J_{y\min} = 1.676 \times 10^{-16}$

Control Configuration 3: $J_{y\min} = 2.652 \times 10^{-17}$

According on the criterion in section 2.2.6.1, the controller in control configuration 3 is better than the control configuration 2 and the controller in control configuration 2 is better than the control configuration 1 in rollover index minimization. Hence we conclude that control configuration 3, which as we recall is the anti- roll bar at the trailer unit, is the best control configuration among the three. It is interesting to realize that in this example application, a single actuator unit did a better job at preventing rollover than a set of two actuators and a set of three actuators. However, it is important to understand that this conclusion is not generic for all vehicle control problems. In fact, the very purpose of this paper is to convey this message that which control configuration is better for the given performance objective is very much dependent on the specific nature of the specific vehicle dynamics and the complicated interactions between the state and control variables. Thus, the procedure outlined in this paper is very useful to be able to make this decision in a logical and systematic way.

2.2.8 A Simulation for Double Lane Change Maneuver

A Closed Loop driver-vehicle-controller system simulation was conducted to investigate the rollover prevention performance using the three control configurations under consideration. In the simulation, a double lane change maneuver is used to represent human drivers in lane following situation and a constant forward velocity of 60km/hr. is assumed. In this simulation, no actuator saturation and actuator dynamics are considered. The controllers using the three control configurations are designed for the situation above. The assigned weighting scalar ρ and the r.m.s. roll torque (control input) which is used to evaluate the control effort are listed below. Note that roll torque (control input) is function of time so the r.m.s. roll torque (control input) is calculated in the simulation period.

Control Configuration 1: $\rho = 1.4 \times 10^{-10}$;
r.m.s roll torque in tractor steering axle = 8944 (Nm);
r.m.s roll torque in tractor drive axle = 257104 (Nm);
r.m.s roll torque in trailer unit = 60200 (Nm);
Total r.m.s roll torque = 326248(Nm).

Control Configuration 2: $\rho = 1.4 \times 10^{-10}$;
r.m.s roll torque in tractor drive axle = 240758 (Nm);
r.m.s roll torque in trailer unit = 71510 (Nm);
Total r.m.s roll torque = 312268 (Nm).

Control Configuration 3: $\rho = 1 \times 10^{-12}$;

r.m.s roll torque in trailer unit = 135825 (Nm) = total r.m.s roll torque.

The steering angle from a driver is shown in Fig. 2. The simulation results of the roll torques (control inputs) for three control configurations are shown in Fig. 3. The rollover index for the tractor steering axle (RI_1), the tractor drive axle (RI_2), and the trailer unit (RI_3) of three control configurations are shown in Fig. 4.

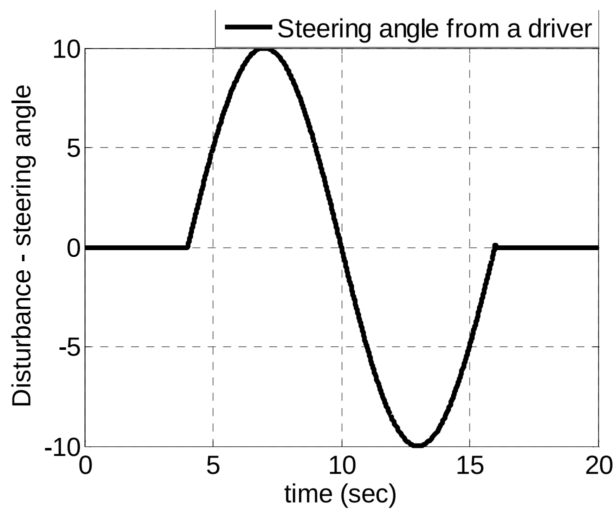
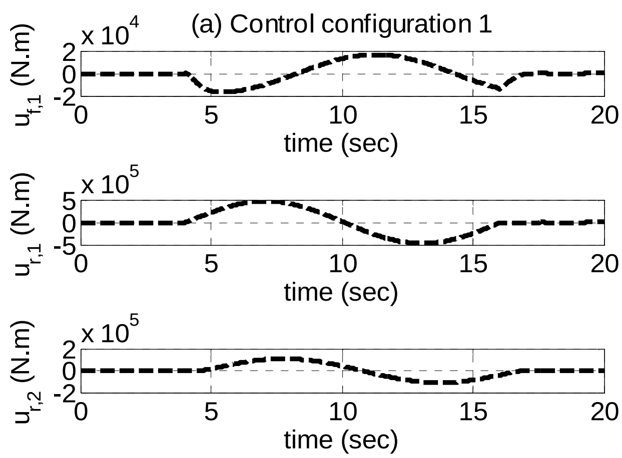


Figure 2 Steering angle from a driver



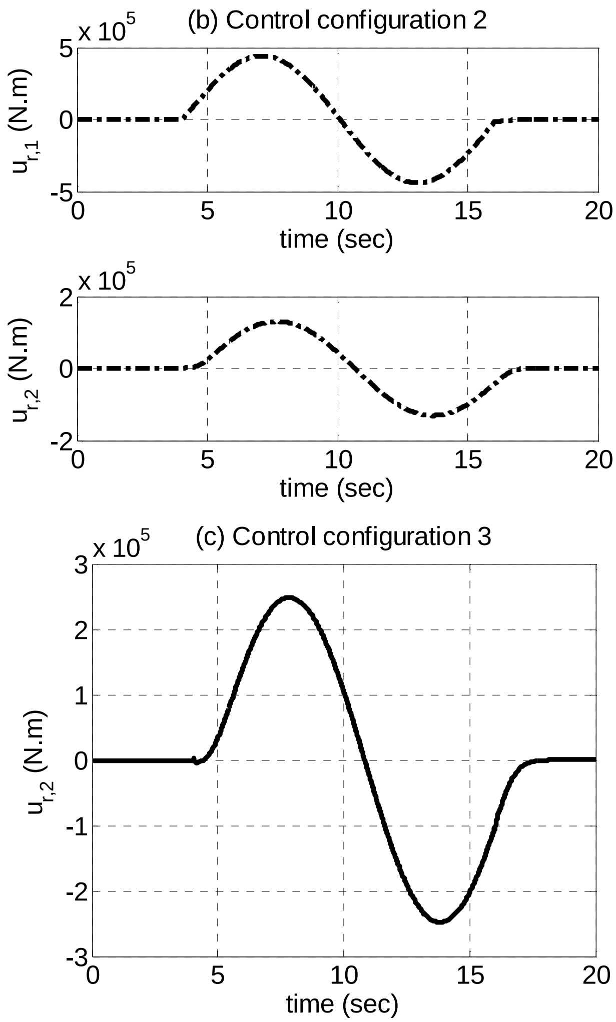
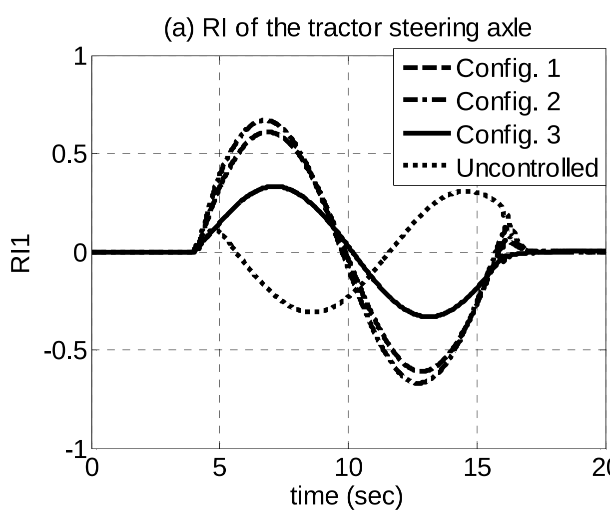


FIGURE 3 CONTROL INPUT (ROLL TORQUE) FOR THREE CONTROL CONFIGURATIONS. (A) CONTROL CONFIGURATION 1 (B) CONTROL CONFIGURATION 2 AND (C) CONTROL CONFIGURATION 3.



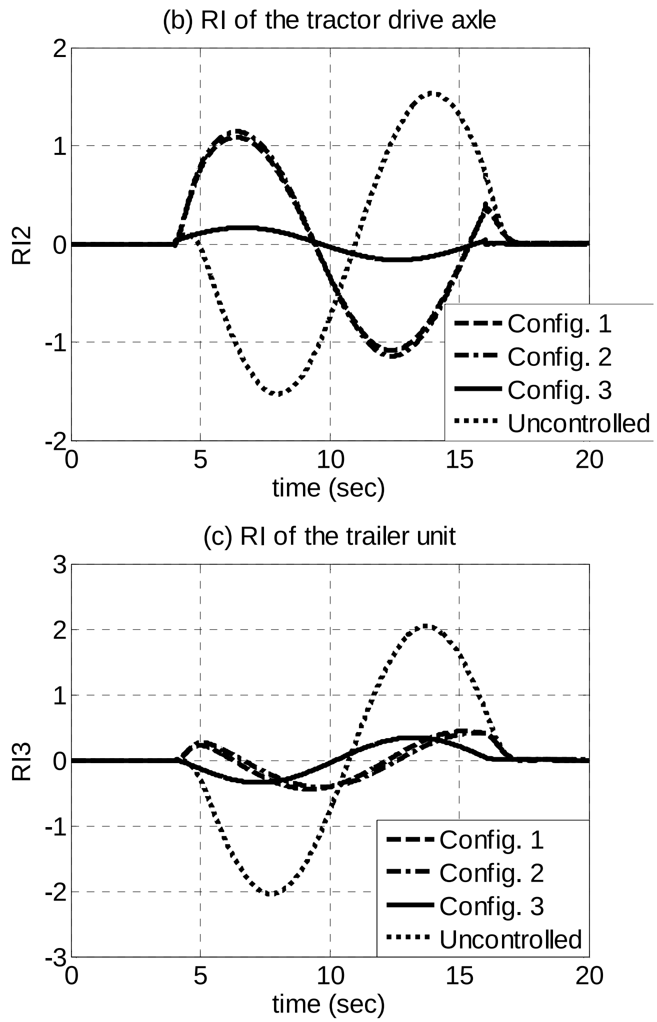


FIGURE 4. ROLLOVER INDICES (RI) FOR THE CONTROL CONFIGURATION 1, 2, AND 3. (A) RI OF THE TRACTOR STEERING AXLE, (B) RI OF THE TRACTOR DRIVE AXLE, AND (C) RI OF THE TRAILER UNIT.

The simulation results show that control configuration 3 can prevent the rollover more efficiently compared to control configuration 1 and 2 under the same situation. Note that actuators are relatively expensive and is desirable to minimize the number of actuators in any engineering system. From that the point of view, the proposed methodology of this paper helps in ranking various control configurations so that only a reduced set of actuators can be recommended thereby providing significant cost savings. In this particular application, it is thus seen that a single actuator corresponding to control configuration 3 is sufficient to avoid rollover. However, there is always a trade-off between allowable control effort and tolerable output regulation cost.

2.2.9 Conclusions

In this paper, a novel controller selection algorithm is presented by extending the popular Linear Quadratic Regulator (LQR) control design method to the ‘state derivative induced output regulation’ problem. In state derivative induced output regulation of linear

state space systems, it is possible to express the output vector in state space realizations with various controller configurations. Thus, when this output is regulated using the performance index minimization technique of LQR framework, the resulting performance index has coupled terms in state and control variables, thereby highlighting the role of controller structure in the output regulation problem. By carefully analyzing this coupling phenomenon, in this paper we present design algorithms that can compare various controller configurations and select one among those for final design. This selection criterion for controller configurations is highly helpful in some applications, where it is possible that higher number of control variables may not necessarily be effective in regulating the output than fewer number of control variables, based on the adverse or beneficial nature of the coupling. The proposed controller selection and design algorithm is applied to rollover prevention for tractor semi-trailer. It is shown that sometimes it is possible that a well designed single controller (actuator) can result in better performance than multiple controllers (actuators) with improper design because of the way the coupling effect interacts with the controller. This in turn can result in considerable savings in actuator costs, controller complexity and control effort.

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APPENDIX I

The elements in matrix C and D for configuration 1 in section 2.2.5.1 are

$$c_{1,1}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,1}-h_{f,1}A_{2,1})+\frac{2k_{f,1}}{m_{f,1}gT};\ c_{1,2}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,2}-h_{f,1}A_{2,2})+\frac{2c_{f,1}}{m_{f,1}gT};$$

$$c_{1,3}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,3}-h_{f,1}A_{2,3});\ c_{1,4}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,4}-h_{f,1}A_{2,4});$$

$$c_{1,5}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,5}-vA_{2,5});\ c_{1,6}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,6}-h_{f,1}A_{2,6}+v_x);$$

$$c_{1,7}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,7}-h_{f,1}A_{2,7});\ c_{1,8}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,8}-h_{f,1}A_{2,8});$$

$$c_{1,9}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,9}-h_{f,1}A_{2,9});\ c_{1,10}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,10}-h_{f,1}A_{2,10});$$

$$c_{1,11}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,11}-h_{f,1}A_{2,11});\ c_{1,12}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,12}-h_{f,1}A_{2,12});$$

$$c_{1,13}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xA_{5,13}-h_{f,1}A_{2,13});\ d_{1,1}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xB_{5,1}-h_{f,1}B_{2,1});$$

$$d_{1,2}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xB_{5,2}-h_{f,1}B_{2,2});\ d_{1,3}=\frac{2m_{s,f,1}h_{R,f,1}}{m_{f,1}gT}(v_xB_{5,3}-h_{f,1}B_{2,3}).$$

$$c_{2,1}=\frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_xA_{5,1}-h_{r,1}A_{4,1});\ c_{2,2}=\frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_xA_{5,2}-h_{r,1}A_{4,2});$$

$$c_{2,3} = \frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_x A_{5,3} - h_{r,1}A_{4,3}) + \frac{2k_{r,1}}{m_{r,1}gT}; c_{2,4} = \frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_x A_{5,4} - h_{r,1}A_{4,4}) + \frac{2c_{r,1}}{m_{r,1}gT};$$

$$c_{2,5} = \frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_x A_{5,5} - h_{r,1}A_{4,5});\ c_{2,6} = \frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_x A_{5,6} - h_{r,1}A_{4,6} + v_x);$$

$$c_{2,7} = \frac{2m_{s,r,1}h_{R,f,1}}{m_{r,1}gT}(v_x A_{5,7} - h_{r,1}A_{4,7});\ c_{2,8} = \frac{2m_{s,r,1}h_{R,f,1}}{m_{r,1}gT}(v_x A_{5,8} - h_{r,1}A_{4,8});$$

$$c_{2,9} = \frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_x A_{5,9} - h_{r,1}A_{4,9});\ c_{2,10} = \frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_x A_{5,10} - h_{r,1}A_{4,10});$$

$$c_{2,11} = \frac{2m_{s,r,1}h_{R,f,1}}{m_{r,1}gT}(v_x A_{5,11} - h_{r,1}A_{4,11});\ c_{2,12} = \frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_x A_{5,12} - h_{r,1}A_{4,12});$$

$$c_{2,13} = \frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_x A_{5,13} - h_{r,1}A_{4,13});\ d_{2,1} = \frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_x B_{5,1} - h_{r,1}B_{4,1});$$

$$d_{2,2} = \frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_x B_{5,2} - h_{r,1}B_{4,2});\ d_{2,3} = \frac{2m_{s,r,1}h_{R,r,1}}{m_{r,1}gT}(v_x B_{5,3} - h_{r,1}B_{4,3}).$$

$$c_{3,1} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,1} - h_{r,2}A_{10,1});\ c_{3,2} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,2} - h_{r,2}A_{10,2});$$

$$c_{3,3} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,3} - h_{r,2}A_{10,3});\ c_{3,4} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,4} - h_{r,2}A_{10,4});$$

$$c_{3,5} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,5} - h_{r,2}A_{10,5});\ c_{3,6} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,6} - h_{r,2}A_{10,6});$$

$$c_{3,7} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,7} - h_{r,2}A_{10,7});\ c_{3,8} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,8} - h_{r,2}A_{10,8});$$

$$c_{3,9} = \frac{2m_{s,f,1}h_{R,r,2}}{m_{f,1}gT}(v_x A_{11,9} - h_{r,2}A_{10,9}) + \frac{2k_{r,2}}{mgT}; c_{3,10} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,10} - h_{r,2}A_{10,10}) + \frac{2c_{r,2}}{m_{r,2}gT};$$

$$c_{3,11} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,11} - h_{r,2}A_{10,11});\ c_{3,12} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,12} - h_{r,2}A_{10,12} + v_x);$$

$$c_{3,13} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x A_{11,13} - h_{r,2}A_{10,13});\ d_{3,1} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x B_{11,1} - h_{r,2}B_{10,1});$$

$$d_{3,2} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x B_{11,2} - h_{r,2}B_{10,2});\ d_{3,3} = \frac{2m_{s,r,2}h_{R,r,2}}{m_{r,2}gT}(v_x B_{11,3} - h_{r,2}B_{10,3}).$$

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